

Actuarial Mathematics (Actuarial Science)

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Background

In the early twentieth century a few US colleges and universities began to offer courses in actuarial mathematics. The number of such schools and enrollments in actuarial courses grew slowly until the 1970s. Federal pension legislation in 1974 dramatically increased the demand for actuaries; the 1988 publication of *Jobs Rated Almanac* listing the job of an actuary as #1 further stimulated student interest, which has remained high as that ranking has remained no lower than fourth. The last quarter of the century consequently witnessed a steady increase in both the number of schools with such courses and the number of schools providing extensive actuarial education programs. By early in the twenty-first century nearly 150 colleges or universities – still a small fraction of the nearly 2500 total – offered some level of actuarial courses or program; over half of these featured extensive programs with multiple courses.

Program types

The knowledge base and course expectations recommended by the professional actuarial organizations evolve over time. In recommending actuarial mathematics courses for various types of programs, we use one subject whose mastery is required for an actuarial designation (see the section **Actuarial exams and designations** below) by either the Casualty Actuarial Society (CAS) or the Society of Actuaries (SOA). [A 3-credit-hour course that is frequently taught as an organized course rather than as individual instruction, with the equivalent of approximately 43 50-minute classes.] As of exam-system changes that took effect in fall 2015, these specialized actuarial courses should be chosen from among the following that relate to the then-current system of exams and VEE requirements described in in the section **Actuarial exams and designations** below:

- a) a course treating the content of CAS Exam 3F and SOA Exam MFE (hereafter denoted CAS/SOA Exam 3F/MFE);
- b) two courses treating the content of either CAS Exam S (beyond basic statistics) or SOA Exam MLC;
- c) two courses treating the content of CAS/SOA Exam 4/C; and
- d) one or two courses covering the VEE requirement called Applied Statistical Methods (for which VEE credit can be obtained in various ways as described in the section **Actuarial exams and designations** below).

Detailed learning objectives and recommended textbooks for each of the exams and VEE subjects mentioned in here can be found through the links in the section **Actuarial exams and designations** below and can be used as the basis for the syllabi for courses covering that material.

While there is a broad range of program types in existence, most undergraduate programs fall into one of the four categories described below; there are also graduate programs, both professional masters programs designed to quickly take graduates of the more basic programs through additional exam material, and Ph.D. programs designed to prepare students for research or academic careers. In the description below of each type of undergraduate program, note that each subsequent level includes all the requirements of the previous level program as well as certain additional course offerings.

Preparatory Program

A Preparatory Program consists only of courses regularly offered at most undergraduate institutions; no specialized actuarial courses need be offered. It provides its students the preparatory background to apply for graduate actuarial programs. For those interested in immediately embarking on an actuarial career, it provides the knowledge needed to make initial headway with the professional exam and designation systems described in the section **Actuarial exams and designations** below. As of early 2014, a Preparatory Program would cover the content of CAS/SOA Exam 1/P, at least the basic one-course probability-based statistics part of CAS Exam S, and the VEE requirement called Economics (for which VEE credit can be obtained in various ways as described in the section **Actuarial exams and designations** below).

The required content of a Preparatory Program is as follows:

1. a course sequence covering single- and multi-variable calculus;
2. a course on linear algebra;
3. a two-course sequence covering calculus-based probability and probability-based statistics; and
4. one course each on introductory microeconomics and macroeconomics.

Some examples of Preparatory Programs follow. The programs listed were chosen (in late 2012) to represent a variety of kinds of schools

Programs. Links to many more actuarial programs can be found on the

(<http://www.soa.org/Education/Resources/actuarial-colleges/actuarial-college-listings-details.aspx>), which uses a classification of program types that differs from what is used here.

- a) Grand Valley State University: <http://www.gvsu.edu/math/>
- b) Harvey Mudd College: <http://www.math.hmc.edu/>
- c) Lewis and Clark University: http://college.lclark.edu/departments/mathematical_sciences/
- d) University of Idaho: <http://www.uidaho.edu/sci/math/Academics/undergraduate/actuarial>
- e) University of Vermont: <http://www.uvm.edu/~cems/mathstat/>

Introductory Program

An Introductory Program contains in addition to the content of a Preparatory Program a course providing a mathematical treatment and analysis of compound interest and its applications to financial transactions, with a semester o

needed to make additional headway with the professional exam and designation systems described in the section **Actuarial exams and designations** below. A course on mathematical interest theory would cover about 75% of the content of CAS/SOA Exam 2/FM. The remaining 25% could be left for students to study on their own, or be included in an expanded version (four or five semester-credit-hours) of the interest-theory course;

- b) Case Western Reserve University: http://stat.case.edu/program_actuarial.shtml
- c) DePauw University: <http://www.depauw.edu/academics/departments-programs/mathematics/studentresources/actuarial/>
- d) Howard University: http://www.coas.howard.edu/mathematics/programs_undergraduate_actuarial.html
- e) Slippery Rock University: http://www.sru.edu/academics/colleges/ches/math/Pages/special_interest_areas.aspx
- f) Texas Christian University: <http://www.math.tcu.edu/actuarial.html>

Intermediate Program

An Intermediate Program contains in addition to the content of an Introductory Program the remaining 25% of the content of CAS/SOA Exam 2/FM plus two specialized actuarial courses as described in the first paragraph of this section **Program types**. These courses apply probability and statistics to actuarial problems and provide the knowledge needed to make additional headway with the professional exam and designation systems described in the section **Actuarial exams and designations** below. Depending on exactly which two specialized actuarial courses were chosen, as of the exam-system changes in fall 2015 the two courses could cover content as follows:

- all the material for CAS/SOA Exam 4/C; or
- all the material beyond basic statistics for CAS Exam S; or
- all the material for SOA Exam MLC; or
- all the material for CAS/SOA Exam 3F/MFE plus some portion of another exam; or
- all the material for VEE Applied Statistics plus possibly some portion of another exam.

In summary, the required content of an Intermediate Program consists of that listed above for an Introductory Program in addition to:

- 7. two specialized actuarial courses.

Some examples of Intermediate Programs follow. The programs listed were chosen (in late 2012) to represent a variety of kinds of schools

Programs. Links to many more actuarial programs can be found on t (<http://www.soa.org/Education/Resources/actuarial-colleges/actuarial-college-listings-details.aspx>), which uses a classification of program types that differs from what is used here.

- a) Aurora University: <http://www.aurora.edu/academics/programs-majors/undergraduate/actuarial-science/index.html#axzz2ASJTnmI5>
- b) Binghamton University: <http://www.math.binghamton.edu/actuary/>
- c) Elizabethtown College: <http://www.etown.edu/depts/math/areas.aspx>
- d) Indiana University Purdue University Indianapolis: <http://www.math.iupui.edu/>
- e) Oregon State University: http://www.math.oregonstate.edu/Actuarial_Science_Program
- f) University of Central Oklahoma: <http://www.math.uco.edu/undergrad/index.html>

Advanced Program

An Advanced Program contains in addition to the content of an Intermediate Program two additional specialized actuarial courses as described in the first paragraph of this section **Program types**. These courses apply probability and statistics to actuarial problems and provide the knowledge needed to make still further headway with the professional exam and designation systems described in the section **Actuarial exams and designations** below. Depending on exactly what specialized actuarial courses are chosen, as of the exam-system changes in fall 2015 the resulting total of four specialized actuarial courses could cover topics as follows:

all the material in both CAS/SOA Exam 4/C and either CAS Exam S or SOA Exam MLC; or
all the material in one of those exams together with half of the other plus the material in CAS/SOA Exam 3F/MFE.

In summary, the required content of an Advanced Program consists of that listed above for an Intermediate Program in addition to:

8. two additional specialized actuarial courses.

Some examples of Advanced Programs follow. The programs listed were chosen (in late 2012) to represent a variety of kinds of schools
Advanced Programs. Links to many more actuaia
of programs (<http://www.soa.org/Education/Resources/actuarial-colleges/actuarial-college-listings-details.aspx>), which uses a classification of program types that differs from what is used here.

- a) Central Washington University: <http://www.cwu.edu/math/>
- b) Florida State University: <http://www.math.fsu.edu/~paris/actmath.math>
- c) Lebanon Valley College: <http://www.lvc.edu/mathematics/program-mathematics.aspx>
- d) Robert Morris University:
<http://www.rmu.edu/Undergraduate/AcademicPrograms/ActuarialScience>
- e) University of Michigan: <http://www.math.lsa.umich.edu/undergrad/actuarial.html>
- f) University of St. Thomas:
<http://www.stthomas.edu/mathematics/actuarialscience/default.html>

Actuarial exams and designations

CAS/SOA exams ~~SEI~~ ~~BO~~

Each organization has its own series of examinations and other procedures leading first to the designation of Associate of that society and then to the designation of Fellow of that society. Each also offers a credential as Chartered Enterprise Risk Analyst. For details, for the SOA see <http://www.soa.org/Education/Exam-Req/default.aspx> and see <http://www.casact.org/admissions/process/> for the CAS. Fortunately for students, most of the initial examinations currently satisfy requirements for both organizations.

until they are employed. However, most employers will expect a student to have passed one or two of the actuarial exams prior to graduation.

The defining feature of an actuarial career is the exam system (<http://www.beanactuary.org/exams/>). Passing actuarial exams is a key part of the path to raises, promotion, and success as an actuary. A minimum passing score on an exam usually equates to solving around 60% of the problems correctly. Each exam is very challenging, and the more mathematical exams the preliminary exams typically have pass rates of only about 35% - 40%. One cause of this low pass rate on exams is

spend at least 100 hours of study for each hour of the exam - beyond the time spent on

- solving mathematics courses like calculus may have the *ability* to pass exams, they will succeed only if they commit to the necessary study hours. It is not sufficient to merely be familiar with the material and/or memorize the important topics. Passing the exam requires deep understanding of the concepts and demands working a very large number of problems in preparation in order to master the material and to develop an efficient exam-taking strategy. The later exams especially require the self-discipline to study independently. Exams can be taken repeatedly until passed.

Many sources provide practice problems and study aids to help students prepare for exams. Various organizations publish study manuals, and students can utilize online and in-person review seminars and workshops (<http://www.soa.org/education/exam-req/resources/edu-sem-workshops.aspx>) (<http://www.casact.org/admissions/studytools/>)

(<http://www.casact.org/admissions/index.cfm?fa=email>). Many past exam questions and solutions are also available online from the SOA and the CAS. A detailed exam syllabus and a list of references can be found on the SOA and CAS websites <http://www.soa.org> and <http://www.casact.org>. Once students thoroughly understand the material, they should focus on taking practice exams. As the examination day approaches, students should take the practice exams under conditions that closely mimic the actual exam situation (time limits, use of approved calculators, etc.).

For many years, the SOA and CAS jointly administered various preliminary exams, ended 31 December 2013, and each organization held separate preliminary exams starting in 2015. For the latest information on preliminary exams, see the CAS website (<http://www.casact.org/admissions/exams/>), the SOA website (<http://www.soa.org/Education/Exam-Req/edu-asa-req.aspx>), or the BeAnActuary.org website (<http://www.beanactuary.org/exams/preliminary/?fa=preliminary-exams>).

CAS Exam 1/SOA Exam P

(<http://www.beanactuary.org/exams/preliminary/?fa=probability-exam>) covers calculus-based probability, primarily the probability contained in a standard two-semester probability and statistics (or mathematical statistics) sequence. The SOA administers this computer-based exam as SOA Exam P; the CAS does not administer a separate CAS Exam 1 but instead gives credit for its Exam 1 to anyone passing SOA Exam P.

CAS Exam 2/SOA Exam FM

(<http://www.beanactuary.org/exams/preliminary/?fa=financial-math-exam>) covers calculus-based interest theory and an introduction to derivative securities. The SOA administers this computer-based exam as SOA Exam FM; the CAS does not administer a separate CAS Exam 2 but instead gives credit for its Exam 2 to anyone passing SOA Exam FM. The interest-theory content might be found in a mathematics course on interest theory and the material on derivative securities might be found in a finance course (although there are sufficient references for students to learn this on their own) or in an expanded version of an interest-theory course; these topics may be studied in either order. Discussions of moving most of derivative securities material to CAS/SOA Exam 3F/MFE were under way in 2015.

It is worth noting that a thorough knowledge of the material from both of these Exams 1/P and 2/FM is assumed for most of the other more advanced exams. Employers expect applicants and new hires to have credit for at least one of these two exams.

CAS Exam 3F/SOA Exam MFE

(<http://www.beanactuary.org/exams/preliminary/?fa=financial-economics-exam>) covers models for financial economics, including material on the mathematics of financial derivatives. The SOA administers this computer-based exam as SOA Exam MFE; the CAS does not administer a separate CAS Exam 3F but instead gives credit for its Exam 3F to anyone passing SOA Exam MFE.

SOA Exam MLC-9(g)10(e)4(of th)-8(e)4(mate)4(r8r.58 Tm[3F)6(to an)-17(y)20(o3(for)6(it)-3(s TJt)-3((

(<http://www.casact.org>), new in fall 2015, covers probability models (stochastic processes, Markov Chains, introductory survival models with applications to life

Students primarily earn VEE credit by passing (with a minimum grade of B-) approved university or college courses, but approved distance-learning courses and exams by other organizations can provide VEE credit as well. Students may apply for VEE credit for approved experiences only after passing two of the SOA/CAS exams, although the experiences may be completed prior to having passed exams. See the CAS or SOA website for information on how to get a course approved for VEE credit (<http://www.soa.org/education/exam-req/course-info/edu-vee-evaluation-process.aspx>) or <http://www.casact.org/admissions/syllabus/index.cfm?fa=vee#asm>) and for the list of all currently approved methods of obtaining VEE credit (<http://www.soa.org/education/exam-req/resources/edu-vee-approved-courses.aspx> or <http://www.casact.org/admissions/syllabus/index.cfm?fa=vee#asm>).

Faculty

Program Coordinator

A successful actuarial program relies heavily on the leadership and guidance of a faculty member serving as Program Coordinator. Advising students, teaching actuarial coursework, and preparing students for the job market are the primary focus of the Program Coordinator.

In building a program, the Coordinator also needs to develop a strong relationship with colleagues in the Economics Department, Finance Department, and School of Business generally, both to ensure that actuarial students have access to appropriate courses in economics, accounting, and finance and to offer opportunities for students in these related disciplines to double major or minor in actuarial mathematics. The Program Coordinator should also be aware of and recommend to students courses outside of mathematics that are related to or useful in the actuarial field, such as economics, accounting, business law, finance, risk management, technical writing, public speaking, computer programming, and spreadsheet and database use.

A Program Coordinator additionally plays an important role as the official contact person to the Society of Actuaries (SOA) and Casualty Actuarial Society (CAS). Updates on exam changes (see the section **Actuarial exams and designations** above),
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An active and strong program connects students to working actuaries through actuarial club activities, on-campus presentations from actuarial firms, and promoting summer internships. The Program Coordinator needs to work directly with campus career services personnel and college relation officers at actuarial companies to create these networking opportunities for students. Furthermore the Program Coordinator needs to ensure that actuarial firms are informed about the campus employment system and career fair events. Program Coordinators should consider organizing an advisory board of actuarial professionals.

Faculty training and resources

The standardized syllabi for the preliminary exams (see the section **Actuarial exams and designations** above) are the natural starting place for actuarial curriculum development. Instructors planning to teach these courses should be familiar with the

<http://www.casact.org/community/academic/index.cfm?fa=ulprog>

Actuarial Educators Network

<http://www.actuarialeducators.org>

Mathematical Association of America

<http://www.maa.org/meetings/joint-mathematics-meetings>

Actuarial Research Conference

Search for this topic on <http://www.soa.org> or <http://www.casact.org>

Education and Research Section of the SOA

<http://www.soa.org/professional-interests/education-and-research/eandr-detail.aspx>

SOA Educational Grants

<http://www.soa.org/education/resources/edu-institution-grant/default.aspx>

CAS Academic Central Program

<http://www.casact.org/membership/join/index.cfm?fa=acadcorr>

CAS Student Central

<http://www.CASstudentcentral.org>

Informing students about careers and job seeking

Career paths

A Program Coordinator (see the section **Faculty** above) has a key responsibility to provide interested students with information on actuarial and other careers for graduates

Each of these fields encounters various challenges of risk management. Insurance firms need actuaries to accurately determine how much money must be set aside or reserved for future claims. Actuaries design systems to price insurance products so as to keep them competitive and profitable. In the federal government, actuaries play important roles in evaluating the financial viability of key social insurance and similar programs; in state government, actuaries play key roles in regulating insurance companies. Pension actuaries manage retirement plans in order to ensure that these plans are financially sound. Actuaries employed in finance or banking may handle hedge funds or develop financial products. Risk consultants work with a wide variety of data, based

market prices, demographic data, or health and longevity data offer a few examples of ever changing information that impacts risks of various kinds. Actuaries need to keep abreast of legislative changes, new regulations, and the current economic climate as well. Analytic reasoning ability and the proficiency to engineer mathematical models are the key skills in each of these actuarial careers.

Career-information resources

To explore the actuarial field and career options, begin with and direct students to the website <http://www.BeAnActuary.org> . This site also includes links to both the CAS and SOA Career Centers. These career centers maintain lists of job postings and information about internships. The CAS offers a free student membership program for students interested in the actuarial career with a dedicated website at <http://www.CASstudentcentral.org> and information and other resources including a student online community, the CAS Curriculum Guide, and case studies. The <http://mathclassifieds.org> , also features job openings of interest.

To learn more about the profession, consult

