Complex Analysis

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Complex analysis is one of the most beautiful as well as useful branches of mathematics.
--Murray R. Spiegel.

Introduction: Complex analysis is indeed a beautiful and useful branch of mathematics. It is one of the classical subjects with most of the main results extending back into the nineteenth century and earlier. Yet, the subject is far from dormant. It is a launching point for many areas of research and it continues to find new areas of applicability, from pure mathematics to applied physics. Many of the giants of mathematics have contributed to the development of complex analysis. Names such as Euler, Cauchy, Gauss, Riemann, and Weierstrass are common occurrences among its list of important results. Technology has opened additional avenues of study using complex analysis, from fractals to color-enhanced methods for visualization. Complex analysis is an important component of the mathematical landscape, unifying many topics from the standard undergraduate curriculum. It can serve as an effective capstone course for the mathematics major and as a stepping stone to independent research or to the pursuit of higher mathematics in graduate school.

by no means universal. Other possible prerequisites include an introduction to proofs course, differential equations, and real analysis. The background of the students directly affects the type of course that can be taught. Fewer mathematical prerequisites allow more students from other majors to take the course. Complex Analysis is particularly well-suited to physics majors. It
nalysis.

The course is also very useful for students planning to go to graduate school in mathematics or applied mathematics. Many graduate programs offer a qualifying exam in real and complex analysis. A solid undergraduate experience can ease the transition to graduate courses and set the stage for more advanced study in a number of interesting and current fields. Students planning to do undergraduate research or a senior capstone may also find a course in Complex Analysis as a useful stepping stone to projects in complex dynamics, differential geometry, or analytic number the2BT8.

Complex analysis is a subject that can serve many roles for different majors and types of students. The material and theems reach into many areas of pure and applied mathematics. At one end of the spectrum, the subject provides a powerful set of tools for dealing with t

other topics and applications. We believe that most of the topics listed should be part of any first course in Complex Analysis.

In providing sample syllabi, we have listed the topics in a reasonable order of presentation, though other orders are certainly possible. We have also suggested the number of lessons that should be devoted to each topic. Our working assumptions were that a typical course would contain forty lessons of 50 minutes each. We have tried to make our core set of topics use 25 to 30 lessons, leaving the remaining lessons for additional topics and assessment.

Following our core set of topics, we present suggestions for three additional courses that each give a different emphasis: the first has the core material plus additional emphasis on more pure mathematics topics, the second has the core plus an introduction to complex dynamics, and the third has the core topics plus an applied emphasis. These syllabi represent a sampling of the myriad possibilities in designing a vibrant undergraduate course in Complex Analysis.

Assumed prerequisite: multivariable calculus. We make particular mention of how to use technology, since technology use depends heavily on the IT infrastructure of the institution.

Sample syllabus for core topics, only

Topics	**Approx. # of lessons**
Complex numbers, complex arithmetic, geometric representation, in the plane (open, closed, connected, bounded, etc.).	3 - 4

Complex functions to include multiple-valued functions and the notion of branches. The geometry of complex functions as mappings from the *z*-

Sample Syllabus for Core Topics, plus an additional emphasis in pure mathematics

Topics	Approx. # of lessons
Complex numbers, complex arithmetic, geometric representation, in the plane (open, closed, connected, bounded, etc.).	3 - 4
Complex functions to include multiple-valued functions and the notion of branches. The geometry of complex functions as mappings from the *z*-plane to the *w*-plane.	3
The typical functions from calculus extended to the complex domain (polynomials, power functions, rational, exponential, logarithmic, trigonometric, and hyperbolic trigonometric).	3 - 4
The theory of differentiation for complex functions. Analytic functions to include the Cauchy-Riemann equations, harmonic functions, and properties of analytic functions.	3 - 4
The theory of integration of complex functions to include the Cauchy-Goursat theorem, Cauchy's Integral formula, and several consequences of the Integral Formula to include Cauchy's Inequality, Liouville's theorem, and the Maximum Modulus Principle.	5 - 6

[A] *Converses of Cauchy's theorem: Morera's theorem, Schwartz ruvie.*

Sample Syllabus for Core Topics plus an additional emphasis in Complex Dynamics

Topics	Approx. # of lessons
Complex numbers, complex arithmetic, geometric representation, in the plane (open, closed, connected, bounded, etc.). *Complex iteration, fixed points, periodic points, pre-periodic points.*	5
Complex functions to include multiple-valued functions and the notion of branches. The geometry of complex functions as mappings from the *z*-plane to the *w*-plane.	3
The typical functions from calculus extended to the complex domain (polynomials, power functions, rational, exponential, logarithmic, trigonometric, and hyperbolic trigonometric).	4

The theory of differentiation for complex functions.
Analytic functions to include the Cauchy-Riemann equations,

Sample Syllabus for Core Topics, plus an emphasis in applied mathematics

Topics	Approx. # of lessons
Complex numbers, complex arithmetic, geometry of complex numbers in the complex plane. Polar and exponential representation of complex numbers,	2
Complex functions to include multiple-valued functions and the notion of branches. The geometry of complex functions as mappings from the z-plane to the w-plane, Möbius transformations. The typical functions from calculus extended to the complex	3

Short Bibliography

Please note that some of the resources listed below were written by the authors of this report.

Standard texts:

> Remark*: The presence of a text on this list is not meant to imply an endorsement of that text, nor is the absence of a particular text from the list meant to be an anti-endorsement. The texts are chosen to illustrate the sorts of texts that support various types of complex analysis courses.*

J. Bak and D. J. Newman, *Complex Analysis*, 3

T. Needham, *Visual Complex Analysis*, Oxford University Press, Oxford, UK, 1997.

Open Source Materials

M. Beck, G. Marchesi, D. Pixton, and L. Salbalka, *A First Course in Complex Analysis*, http://math.sfsu.edu/beck/complex.html

G. Cain, *Complex Analysis*, http://people.math.gatech.edu/~cain/winter99/complex.html

http://www.sagemath.org

Websites and Applets

Applets by Jim Rolf that allow for exploration of complex functions, iteration, Julia sets, and the Mandelbrot set can be found here.

This collection of applets makes it possible for students to explore the behavior of a variety of