Discrete Mathematics & Combinatorics

Alan Tucker, SUNY Stony Brook (chair)
Doug Baldwin, SIGCE/Discrete and SUNY Geneseo
Karen Collins, Wesleyan University & SIAM
Susanna Epp, DePaul University
Diana Thomas, Montclair State University

This report is divided in two parts, the first and major part about Discrete Mathematics courses, typically a lower-division offering, and the second part about Combinatorics courses, typically an upper-division offering.

I. Discrete Mathematics Courses

Audience for the course: Students majoring in computer science form the primary audience for Discrete Mathematics courses; however, at some institutions the course also serves as a transition to abstract mathematics for mathematics and mathematics education majors and is sometimes taken by students majoring in other technical subjects. A major challenge for the course, addressed below, is how to accommodate both computer science majors and mathematics majors, given that the mathematical backgrounds of these two groups may differ. For information about separate transition courses that use discrete mathematics topics, see this Guide's report on transition courses.

History of the course. For computer science students, discrete mathematics is intended to provide a mathematical foundation for further study of some or all of the following areas: programming languages, computer organization, database theory, data structures, analysis of algorithms, computability, formal methods in software engineering, and network security including cryptography. This course originated in the first ACM undergraduate curriculum recommendations in 1968. By 1986 there was a movement to broaden the audience for the course, and an MAA report from a committee consisting of mathematicians and computer scientists recommended that "discrete mathematics should be part of the first two years of the

the introductory Discrete Mathematics course out of mathematics and into computer science. Thus mathematics departments wishing to develop or retain Discrete Mathematics as a service for computer science are urged to consult closely with faculty in computer science in designing or updating their offerings. This is especially helpful in determining which computer science applications to include in the Discrete Mathematics course and which applications the computer science department is happy to take charge of itself. It is also desirable for mathematics departments to ensure that course instructors have enough knowledge of computer science to be able to indicate relationships between the course topics and future computer science topics. As is the case for most mathematics service courses, the lengthy list of topics desired by a client department can create tension between including all the material and ensuring that students attain the proficiency desired by the department being served. Another reason for communication between mathematics and computer science faculty is to alleviate this tension.

Prerequisites. The preparation of students for Discrete Mathematics courses varies significantly among institutions. Prerequisites are typically at the level of precalculus but vary from two semesters of calculus to college algebra (or the equivalent at the high school level), and some students' algebra skills are rather weak. Thus decisions about the appropriate balance between breadth, depth, and what can be expected from a one-semester course will vary from one institution to another.

Cognitive learning goals. Discrete Mathematics courses have the following primary cognitive learning goals. These goals are developed in almost all the course topics.

Familiarity with the role logical reasoning plays in mathematics: Students should

Pedagogy. Because many of the cognitive learning goals aim to improve students' ability to evaluate mathematical statements, justify their truth or falsity, and express themselves clearly and precisely, a chief teaching recommendation is to ensure students' active participation and give them frequent feedback on their work.

Addressing studentsô varied backgrounds: Students come to a Discrete Mathematics course with a variety of backgrounds and abilities. All the cognitive learning goals are important, but the level at which students will achieve them will vary from student to student and from institution to institution. Instructors can address the fact that students absorb ideas and develop new skills at different rates by incorporating review of basic logical principles when introducing new topics and by mixing, throughout the course, straight-forward, concrete problems with ones that are more challenging and abstract. The majority of students coming into a Discrete Mathematics course, even those who have studied calculus, will have had little or no previous experience with the modes of thought listed as cognitive learning goals. For them, one course is just a start on the road to deep understanding. Follow-up courses will need to extend and reinforce the mental abilities they develop in this course.

Syllabus

1. ***Basic Logic and Introduction to Proof***

A>DC BT F2 12 Tf1 0 1 2.010.31 to Proof

B. *Functions*. Definitions of relations and functions; examples of functions defined on general sets; one-to-one, onto, and inverse functions; composition of functions; cardinality.

C.

Within the general context of solving problems from first principles, there are usually several different approaches for solving a combinatorial problem; for example, breaking the whole problem into more tractable subparts; solving an appropriate subcase and extending the analysis to other subcases; solving a complementary problem; or finding a way to re-state the problem in terms of a previously solved problem. This leads to the second main cognitive goal for such courses: facility with such multiple approaches in problem-solving. First, students need to be ‘desolve some of them; Fibonacci numbers and their applications;

Optional Topics: Generating functions, partitions, Polya's enumeration formula, combinatorial designs, Ramsey theory.

2. ***Graph Theory***
   A. *Basic properties of graphs.* Paths, circuits, and connectedness; isomorphism; planarity and dual graphs; digraphs.
   B. *Trees.* Basic properties; spanning trees; searching problems. Comments: trees are particularly important for computer science students.
   C. *Graph coloring.* Chromatic number; map coloring, some coloring theorems.
   D. *Eulerian and Hamiltonian circuits.* Euler circuit theorem; existence and non-existence of Hamiltonian circuits.

   Optional Topics: tournaments; matching; network flows; matroids.

Prerequisites are a matter of mathematical maturity rather than content knowledge. Often, a sophomore mathematics course, such as linear algebra, is required as a proxy for this maturity prerequisite.

A two-course sequence has a natural structure, covering enumerative material in one course and graph theory in the other.

Applications are discussed primarily in the graph theory part of the course. However, specifics of most applications are usually a bit messy, and so most applications are presented in a sketchy way without concrete examples. However, it should be emphasized that this course is valuable

Resources:

Remark*: The presence of a text on this list is not meant to imply an endorsement of that text, nor is the absence of a particular text from the list meant to be an anti-endorsement. The texts are chosen to illustrate the sorts of texts that support various types of Discrete Mathematics and combinatorial mathematics courses. Please note that some of the books listed below were written by the authors of this report.*

Possible texts for *Discrete Mathematics Courses*

1. Bender, Edward A. and S. Gill Williamson, *Lectures in Discrete Mathematics*.

   This book is freely available online.

2. Chartrand, Gary and Ping Zhang, *Discrete Mathematics*. Waveland, 2011.

   This book has a full chapter on planar graphs and graph colorings.

3. Ensley, Douglas E. and J. Winston Crawley, *Discrete Mathematics: Mathematical Reasoning and Proof with Puzzles, Patterns, and Games*, Wiley, 2005.

   As indicated in the subtitle, this book uses puzzles and games as a basis for exploring the standard discrete mathematical topics.

4. Epp, Susanna S., *Discrete Mathematics with Applications*, Cengage Learning, 4th edition 2011.

   This is a widely used text. A briefer version is available as *Discrete Mathematics: An Introduction to Mathematical Reasoning.*

5. Ferland, Kevin, *Discrete Mathematics*, Cengage Learning, 2009.

6. Graham, Ronald L. and Donald Knuth and Oren Patashnik, *Concrete Mathematics: A Foundation for Computer Science*, Addison-Wesley Professional, 1994.

   While more advanced than for a typical lower-division course, this book is valuable for reference and to recommend to good students.

7. Gries, David and Fred Schneider, *A Logical Approach to Discrete Math,* Springer, 1993.

8. Hunter, David J., *Essentials of Discrete Mathematics,* 2nd Ed, Jones and Bartlett, 2011.

   Chapters in this text are labeled by the modes of thinking they promote.

9. Johnsonbaugh, Richard, *Discrete Mathematics,* 7th Ed. Pearson, 2008.

This is a significantly updated version of one of the original texts for a Discrete Mathematics course.

10. Rosen, Kenneth H., *Discrete Mathematics and Its Applications*, 7th Ed., McGraw-Hill, 2011.

This encyclopedic text is widely used, especially in computer science departments. The separately available *Student's Solutions Guide for Discrete Mathematics and Its Applications* is popular.