Mathematical Biology

A proposed update for the 2015 CUPM Guide to Undergraduate Programs in Mathematics Sciences

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Overview

The confluence of mathematics and biology is central to scientific advancement in the 21st century. Challenges as diverse as global climate change, pharmaceutical design, emergent diseases, and genomics-age medicine all require scientists and mathematicians with expertise in both fields.

An integrated view of life science education was heralded in the NRC Bio2010 report [BIO2010: Transforming Undergraduate Education for Future Research Biologists (2003). The National Academies Press, Washington, DC] and explicitly argued for incorporation of a diversity of mathematical topics throughout biology courses and not simply isolated in the mathematics and statistics courses undergraduates in the life sciences were being required to take. Thus, the state of biology education with respect to mathematics is largely settled. The state of mathematics education with respect to biology however, is harder to pin down.

This committee is especially cognizant of the need to present recommendations that respect departmental autonomy, so as to accommodate a variety of institutional attitudes toward interdisciplinary curricula as well as departmental (and interdepartmental) staffing challenges. To that end, we outline several foundational courses, indicate some directions for more advanced undergraduate study, and present a list of fundamental mathematical competencies. We conclude with some recommendations regarding biological competencies.

Throughout, we have in mind as our audience departments of mathematics at liberal arts colleges, research and comprehensive universities, and community colleges. Such departments may wish to construct a new major or minor, a concentration within an existing major or an interdisciplinary concentration attached to an existing major. We encourage collaboration and cooperation with other departments that share these interests. Our hope is that by presenting competencies rather than prescribed mathematical content, we will provide the flexibility needed for multiple routes to success, based on local capabilities. For the same reason, we have steered away from offering specific examples of successful programs. This is especially important given the enormous breadth of biology as a discipline and the myriad of possibilities for institutions to design programs that would address their needs and objectives effectively.

Student Audience

Mathematical biology programs will benefit students with a range of career goals, educational interests, and mathematical backgrounds. The motivation for some students to pursue mathematical biology education will relate to an interest in medical and health-related directions. The 2009 report from the American Association of Medical Colleges and Howard Hughes Institute, ``Scientific Foundations for Future Physicians," (American Association of Medical

Bioinformatics; Bioengineering; Mathematical Biology; Biostatistics; Computational Biology; Computational Neuroscience; GBCB (Genetics, Bioinformatics, and Computational Biology). Some programs are interdisciplinary in the sense that the students are officially in a mathematics graduate program, but their research is located in another department. Moreover, graduate programs in many biology subdisciplines may welcome students with the kind of preparation we have described.

Prerequisite Skills and Knowledge

A central question for any interdisciplinary program is how much foundational knowledge is required from each discipline. For a mathematical biology program, we assume that during the course of study students will complete some standard courses from the mathematics and biology majors. On the mathematics side, we assume that students will gain competence in logical reasoning, basic methods of proof, and the standard topics associated with the typical mathematics major: Calculus, Linear Algebra, and ODEs. The choice of courses on the biology side depends on the biology subdiscipline of interest. One or two introductory courses required for the biology major will be an appropriate start, fo4 (a)4 (n a i)-6 o a2 (y)-2 (dur)3 ets dururspp depatvscion fou

(A1) Recognize, describe, and give examples of how nonlinear systems differ from linear ones and the process of local linearization for nonlinear systems, and be able to justify the use of a nonlinear system versus a linear system for modeling.
(A2) Recognize, describe, and give examples of the concepts of an optimum, constrained vs. unconstrained optimization, and the choice of an objective function for optimization purposes.

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(A3) Be able to find and analyze stability of equilibria of dynamical systems.
(A4) Be able to analyze spatial and spatiotemporal aspects of biological and physical systems.

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- (SLOM1, SLOM2, SLOM3, SLOM5, SLOM6) modeling terminology, such as discrete and continuous, deterministic and stochastic, and various types of mathematical expressions (e.g., mass-action, exponential growth and decay, logistic growth, and Michaelis

courses as part of their academic offerings, and in many cases the courses may be cross-listed as upper-level undergraduate/low-level graduate courses. For each of the courses listed, the topics we include are those that we believe can be particularly useful for students in mathematical and computational biology. Depending on institutional needs and preferences, the courses we present here may not include all suggested topics, and some of the listed topics may be omitted. In addition, there are many courses typically not taught in math units that could be appropriate for undergraduate math bio students. Our general advice is that students consider taking as many courses from the list below as feasible and as many courses in biology as possible, based on interest and advisor recommendations.

Modeling applied to biological systems -- A course focusing on (1) the nature of information that can be obtained about various biological systems and how this information factors into the development of corresponding mathematical models, and (2) methods to draw, communicate, and defend reasoned conclusions about the underlying biological systems using such models. **Cognitive Recommendations Addressed:** *CR1, CR2, CR3, CR4.* **Learning Outcomes Addressed:** *SLOM1, SLOM2, SLOM3, SLOM4, SLOM5, SLOM6, SLOM7, SLOM8, SLOM9, SLON1, SLON3, SLON4, SLON5, SLOD1, SLOD2, SLOA1, SLOA4.*

Some possible topics for consideration at the undergraduate level:

- dimensional analysis, scaling, and proportionality
- non-dimensionalization of a mathematical model
- fitting data with least squares
- modeling biological systems with difference equations and mathematical analysis of difference equations
- linear biological models such as age-structured population dynamics models, Leslie matrices
- Markov chains and their use in biological modeling
- chemical reactions occurring within biological systems, especially within cells, and the roles of substrates, enzymes, and products within these reactions
- ordinary differential equations that represent a system of reactions using the law of mass action, and the roles of reaction rates in such models
- Gillespie’s algorithm for systems of reactions
- the relationship between deterministic and stochastic representations of chemical reaction systems
- the quasi-steady state assumption
- Michaelis-Menten rate law and Hill functions and their roles in modeling biochemical reaction systems
- positive and negative feedback effects within dynamic models
- concepts of game theory such as two-player games, payoff matrices, evolutionary stable states, Nash equilibria and their relevance to biological s09.1 (tdm (r)5 l)-2 (s)-y(s)1 2 p(d)2 (th)2 /TT1 1 d

Cognitive Recommendations Addressed: *CR1, CR2, CR3.* **Learning Outcomes Addressed:** *SLOM1, SLOM2, SLOM4, SLOM5, SLOM7, SLOM8, SLOM9, SLON3, SLON4, SLOA1, SLOA3.*

Some possible topics for consideration at the undergraduate level:

- difference equations vs differential equations (maps vs flows)
- modeling the evolution of complex systems using dynamical systems; use of continuous time vs. discrete time; use of continuous-space vs. finite-state models
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- Dirichlet and Neumann boundary conditions, linking these with separation of variables, and Duhamel’s principle
- Fourier series and associated concepts (orthogonality, truncation, concepts associated with convergence, periodic extension of a function, sine and cosine series)
- uniqueness theorem for the wave equation
- d’Alembert’s solution for the wave equation IVP and the method of images
- principle of causality, domain of dependence, and domain of influence for the wave equation
- Laplace’s equation in various coordinate systems and solutions of Laplace’s equation on corresponding domains

Stochastic Modeling -- A course devoted to the theory (us)mAvp (a)4 (l)-2 of cutionso3 (i)-2 (e)4 (s)-1 ( )-10 (a)4

- Biological systems and networks: the immune system, the brain, the circulatory and respiratory systems, gene regulatory networks, ecosystems, foodwebs. Systems biology takes a “holistic” approach to study interactions among the components of a biological system and the impact of those interactions on the overall function and behavior of the system.
- Population biology: How populations grow and the main forms of interati(cw)2 (a)402 (i)8 Tm(Tf-4.53 -1.1

- SIAM Activity Group on the Life Sciences: https://www.siam.org/membership/Activity-Groups/detail/life-sciences
- Intercollegiate Biomathemathics Alliance (IBA):
- https://about.illinoisstate.edu/iba/
- QUBES: Quantitative Undergraduate Biology Education: https://qubeshub.org

B. Online References

These include links to publications or organizations especially concerned with mathematical biology curriculum development:

- Bio2010: https://www.nap.edu/catalog/10497/bio2010-transforming-undergraduate-education-for-future-research-biologists
- Math/Bio 2010: https://www.maa.org/press/maa-reviews/math-bio-2010-linking-undergraduate-disciplines
- HHMI/AAMC Scientific Foundations for Preparing Future Physicians: