Mathematical Modeling

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Introduction

> *"The muscles of mathematics are connected to the bones of experimental science by the tendons of mathematical modeling."* Glenn Ledder [3].

The pursuit of abstract mathematical knowledge for its own sake holds a venerable and well-deserved position among the activities worthy of an educated person. However, as suggested by Ledder's metaphor in the preface to his textbook on mathematical applications to biology, the field of mathematics connects to the full richness of human experience through the process of mathematical modeling. Much of the relevance and value to be found in the study of applied mathematics in the 21st Century follows from its ability to capture the structure of observable experiences and, consequently, to support our efforts to solve a wide array of real and meaningful problems.

Because mathematical modeling plays a vital role in delivering the power of mathematics to the needs of science, commerce, politics, and so many other areas of human interest, undergraduate programs in mathematics should seek to provide intentional, substantive learning opportunities for their students in the experience of mathematical modeling. However, the manner in which these experiences are situated in the curriculum may vary considerably, depending on the mission, size, resources, and setting of each institution. This report identifies considerations that would be common to the teaching and learning of mathematical modeling regardless of implementation, outlines several viable ways to situate mathematical modeling within the undergraduate program (not all of which entail stand-alone courses in modeling), and provides resources to assist with the development of modeling curricula.

Definition of Mathematical Modeling

Mathematical modeling is best understood as an active process, rather than a static object of study. In practice, modeling entails a systematic approach to problem solving that brings the techniques and structures of mathematics to bear in an effort to describe, understand, and make predictions about a wide range of empirical phenomena. Genuine mathematical modeling is dynamic and iterative in a way that isolated "story problems" or brief applications (often included as short examples near the end of each section in a mathematics textbook) are not. A recent report from a joint SIAM-NSF workshop drew a similar distinction between "mathematical modeling" and "mathematical models":

> Mathematical modeling is an abstract and/or computational approach to the scientific method, where hypotheses are made in the form of mathematical statements (or mathematical models), which are then used to make predictions and/or decisions. The quality of these models is then examined as part of the verification process, and the entire cycle repeats as improvements and adjustments to the model are made. The teaching of models, by contrast, is simply a presentation of the final product, and does not provide many insights into the process or the understanding gleaned from it….It's like the difference between painting a picture and looking at paintings in a museum. [4]

Although there is surely something to be gained by introducing students to significant and interesting mathematical *models* at points throughout the undergraduate mathematics curriculum, this report is primarily concerned with the teaching and learning of *modeling* in the undergraduate mathematics curriculum. By definition and design this should entail an iterative process in which students establish assumptions, develop a mathematical structure consistent with those assumptions, produce hypotheses or conjectures that are generated by the mathematical structure, test the hypotheses against empirical evidence, and then revise/refine the model accordingly.

The physical sciences and engineering fields have long served as the epicenter for mathematical modeling in the undergraduate curriculum. This is not at all surprising in light of the central role that mathematical language has played in these fields, dating at least as far back as the scientific revolution. However, in the past century--

State Standards, call for students to develop proficiency in mathematical modeling, which will increase the number of undergraduates that arrive at universities with some modeling experience [2].

The United States Military Academy, for example, features an integrated sequence of four mathematics courses (Discrete Dynamical Systems and Introduction to Calculus, Calculus I, Calculus II, and Probability & Statistics), throughout which students engage in mathematical modeling [1]. Significant modeling experiences are often presented in the form of Interdisciplinary Lively Application Projects (ILAPs), many of which are available in a volume published by the MAA.

Macalester College serves as an example of a relatively small liberal arts college that offers a modeling-intensive track (within the Mathematics Major) in Applied Mathematics and Statistics. Students in the Macalester Mathematics Program are introduced to mathematical modeling early, often in their first-year courses, and an orientation toward applications remains present in many

Course Objectives. Courses in mathematical modeling should typically strive to:

1. introduce students to the elements of the mathematical modeling process;
2. present application-driven mathematics motivated by problems from within and outside mathematics;
3. exemplify the value of mathematics in problem solving; and
4. demonstrate connections among different mathematical topics.

Effectively modeling courses should seek student learning outcomes that are consistent with the following learning goals.
Cognitive Learning Goals. Students who successfully complete a course in mathematical modeling should be able to:

1. translate everyday situations into mathematical statements (models) which can be solved/analyzed, validated, and interpreted in context;
2. identify assumptions which are consistent with the context of the problem and which in turn shape and define the mathematical characterization of the problem;
3. revise and improve mathematical models so that they will better correspond to empirical information and/or will support more realistic assumptions;
4. assess the validity and accuracy of their approach relative to what the problem requires;
5. work as members of a team toward a common goal, and
6. communicate mathematics in both oral and written form to a broad mathematical and lay audience, including the "end users" of a modeling problem, who may be utterly unfamiliar with the mathematics used.

Pedagogy

Courses focused on mathematical modeling are particularly well suited to a teaching and learning environment that is centered around open-ended inquiry and experimentation. Bl, asTm[(6. s0( )] TJET

Tips for Modeling in the Classroom

The *process* of mathematical modeling is best learned in the context of actual modeling problems, not as a separate topic in the syllabus. In a typical stand-alone course, students should be given the opportunity to explore many different modeling scenarios. This section contains advice on how to implement mathematical modeling in a stand-alone course or as part of a course.

Use a problem-oriented approach: It is valuable for students to read an authentic open-ended problem and suggest answers to questions like: “What information do we need to address the problem?” “Is the needed information available somewhere?” “Are there assumptions we should make?” “What mathematics may be useful to address the problem?” Once the students have some of these answers, they can begin to formulate a model based on mathematics they know.

Build on students' ideas: Once students formulate models and use them to answer questions, the doors are opened to discussions of concepts that some students offered but were unfamiliar to others. New material relevant to the problem may be introduced at this point. Some problems might be revisited after students have learned new material that could suggest alternative models for the same situation.

Work in teams: Teamwork promotes communication and cross-fertilization of ideas, mirrors the environment in which this work is typically carried out in the field, and fosters an enthusiastic atmosphere that will make the course more interesting for everyone involved.

Let students be creative: Because modeling is a creative process, it is a good idea to encourage creativity in their assumptions and models. This also promotes developing criteria for validating model conclusions and emphasizes the cyclic nature of the process (where models are revised and improved incrementally). Here it is also important not to steer students into a predetermined model that the instructor may have in mind. Allow students enough time to grapple with the problem.

Describe the stages of modeling: Once students have experienced the modeling process through examples, it is a good idea to systematically go over the modeling cycle and the elements that each stage might entail.

Let students write: Modeling projects, especially those that culminate with some sort of “deliverable,” require writing in a way that many undergraduate mathematics courses do not. These situations typically call for a careful mix of technical language along with descriptive--or

Choose situations to model from multiple sources: Traditional modeling situations tend to come from science (astronomy, physics, ecology, etc.) but students are also interested in other sources like truth in advertisement (claims made in ads), social networks, political science, economics, health, local communities, sports, etc. The more students relate to the topic, the more engaged they will be.

A faculty member may feel more comfortable introducing mathematical models and then

Finding and Choosing Modeling Problems

Although aimed at a younger audience, SIAM's Moody's Math Challenge has examples of excellent modeling problems for the advanced high school junior or senior. These problems could easily be made more sophisticated for an undergraduate audience and can serve as a great modeling starting point. The archive that contains all of the past problems can be found at the link below.

http://m3challenge.siam.org/about/archives/

The Mathematical Contest in Modeling (MCM) is a contest where teams of undergraduate students solve real world problems using mathematical techniques. The teams are challenged to tackle an open ended problem from beginning to end and craft an elegant summary of their result. Past problems can be found in the archive below and solutions are published in the UMAP Journal.

http://www.comap.com/undergraduate/contests/matrix/index.html

The archive of problems from Moody's Math Challenge and MCM have been vetted by teachers, industry professionals and professors and are an excellent resource for instructors seeking examples of large scale mathematical modeling problems.

Modeling problep " d lar

to state a model. Once they decide, this information must be researched and found. Different models (deterministic or statistical) are possible and the model conclusions are open to interpretation and validation.

Special Considerations

Modeling Across the Curriculum versus a Stand-Alone Modeling Course

Stand-alone courses in mathematical modeling are well-positioned to provide the time, depth, colleges and universities that prepare future teachers will want to ensure that their mathematics curriculum is configured in such a way that pre-service teachers gain meaningful experience with modeling and applications of mathematics, either through modeling activities integrated into the core mathematics curriculum, through a dedicated course focused on mathematical modeling, or both.

References

[1] Arney, David, and Snook, Kathleen, "Mathematical Modeling in the USMA Curriculum," October 3, 2014.

[2] Common Core State Standards Initiative, "Standards for Mathematical Practice", 2014.

[3] Ledder, Glenn, *Mathematics for the Life Sciences*

11. Fraga, R., *War Stories from Applied Math: Undergraduate Consultancy Projects*, MAA Notes, 2006.

12. Society for Industrial and Applied Mathematics (SIAM), Moody's Mega Math Challenge Website.

Site contains past problems and supplemental resources associated with the Moody's Mega Math Challenge high school modeling contest. Many of the problems and ideas can be adapted to an undergraduate audience.