Number Theory

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1 Student audience

Number theory is an attractive way to combine deep mathematics with familiar concrete objects and is thus an important course for all mathematics students: for \straight" mathematics majors, for pre-service high school teachers, and for students who are preparing to go to graduate school. In ad-

3 Cognitive goals

Analytical and critical thinking are at the core of any Number Theory course. Students' familiarity with the positive integers means that they can begin to apply the results of the course quickly. At the same time, the existence of apparently rare but easily veri ed counterexamples (e.g. Fermat primes, pseudoprimes) forces the student to acknowledge that patterns may break down. Number theory is also famous for having a large number of problems whose di culty is, shall we say, not obvious to discern on rst reading. (This

Beyond these ideas, Number Theory courses tend to fall into two main types, which a ects what additional topics are studied in the course:

Type I{ A course that focuses largely on developing students' proof-writing skills. Within this type there are two subtypes:

- **Ia{** The course is the students' rst serious introduction to proof-writing; such a course usually has calculus and/or linear algebra as its prerequisites.
  Additional topics: methods of proof (direct proof, proof by contrapositive, and proof by induction) and they help students learn to create their own proofs.
- **Ib{** The course has introduction to proof-writing as a prerequisite but focuses on further developing those skills; in this way the course is viewed as a preparation for abstract algebra and/or real analysis, or taught in parallel with them.
  Additional topics: quadratic residues, the law of quadratic reciprocity, and primitive roots.

Type II{ A course that focuses largely on gaining greater depth in mathematics, with Abstract Algebra I as a prerequisite.
Additional topics: These courses have varying syllabi that depend on the instructor's expertise and interests. Some seem to delve further into algebraic Number Theory; others develop topics related to analytic Number Theory.

Additional topics: applications of congruences (cryptography, primality testing), distribution of primes, perfect numbers, continued fractions, Gaussian integers, Diophantine equations. *More advanced topics:* number-theoretic functions, p-adic numbers, Bernoulli numbers and Bernoulli polynomials.

As a future issue, the nature of computation in the course is unresolved. There are three natural modes: pen and paper, using computer algebra systems and writing simple programs directly. Computers can be used to test conjectures generated in class, or to look at certain types of numbers statistically. For example is it common or uncommon for a natural number to be expressible as a sum of two squares? Increasingly, computer algebra systems are being used to implement the RSA encryption process or to carry out some other application requiring lengthy computations.

5 Trolling for information and ideas

With the permission of Gavin LaRose, webmaster for the Project NeXT mailing Lists, the chair of the committee asked members of the six Project NeXT lists whether they would be willing to answer the following three questions:

1. How is undergraduate number theory taught in your institution? (That is: syllabus, textbook, goals, prerequisite, target audience, technology used, writing requirements, etc. Speci c syllabi are welcome!)

2. How does your undergraduate Number Theory course t into your department's expectations for the undergraduate math major?

3. In an ideal educational environment for your institution, how would the Number Theory course di er from the one you have now?

The Project NExT survey led to about 30 responses, some from colleagues of the NExT-ers who had been contacted. [1] The committee would like to acknowlege the help of those who responded. Here is an alphabetical list; those who gave particularly detailed responses are noted by an asterisk.

Justin Brown (Olivet Nazarene), Du Campbell (Hendrix), David Cox (Anherst), Harold Diamond (UIUC), Geo rey Dietz (Gannon), Karrolyne Fogel (Cal Lutheran), Chris French (Grinnell), Darren Glass (Gettysburg), Bonnie Gold (Monmouth), Joshua Holden (Rose-Hulman), Marty Isaacs (Wisconsin), Paul Jenkins (BYU), Eric Kahn (Bloomsburg), Erika King (Hobart and William Smith), Matt Koetz (Nazareth), Carl Lutzer (RIT), Ted Mahavier (Lamar), David Murphy (Hillsdale), Jennifer Paulhus (Grinnell), Tommy Ratli (Wheaton), Thomas Roby (Connecticut), Doug Shaw (UNI), Thomas Sibley (St. John's), Michael Starbird (Texas), Jeffrey Stopple (UCSB), Chris Storm (Adelphi), Wayne Tarrant (Wingate), Steve Ullom (UIUC), Carolyn Yackel (Mercer), Huiya Yan (Wisconsin-La Crosse), Andrew Yang (Dartmouth), Nicholas Zoller (Southern Nazarene).

6 Dreaming

The most persistent dream among our respondents was for math departments and undergraduate programs to be large enough that Number Theory courses

[1] The chair sent a similar, more narrowly focused, email to the faculty on the number theory mailing list at UIUC. He did not receive much useful material from this list.

could be offered more often and with a greater variety.

Instructors often mentioned the wide range of differences in background for students from their own program. An advanced undergraduate mathematics course would be richer if one could assume a stronger background in other courses. In particular, if students know about groups, rings and fields already, much of the basics of modular arithmetic could be just briefly reviewed. Several people expressed a desire to get far enough to talk about elliptic curves.

7 Bibliography

What follows is a list of the textbooks used by our highly unrandom and self-selected group of Project NExT alumni mentioned above. The integers indicate the number of times a text was listed either as primary or supplemental by one of the respondents.

> Remark: *The presence of a text on this list is not meant to imply an endorsement of that text, nor is the absence of a particular text from the list meant to be an anti-endorsement. Please note that some of the books listed below were written by respondents of the poll.*

1. Agnew, Jeanne, *Explorations in Number Theory*. Brooks-Cole Publishing, Co, 1972. (1)

2. Anderson, James A and James M. Bell, *Number Theory with Applications*, Prentice Hall, 1997. (1)

3. Andrews, George E, *Number Theory*, Courier Corporation, 2012. (2)

4. Burton, David, *Elementary Number Theory*, McGraw Hill Education, 2010. (4)

5. Dudley, Underwood, *A Guide to Elementary Number Theory*, MAA Publications, 2009. (4)

6. Edwards, Harold M. *Fermat's Last Theorem: A Genetic Introduction to Algebraic Number Theory*, Springer Science and Business Media, 2000. (1)

7. Jones, Gareth A. and Josephine M. Jones, *Elementary Number Theory*, Springer Science and Business Media, 2012. (2)

8. Marshall, David C., Edward Odell, and Michael Starbird, *Number Theory through Inquiry*, MAA Textbooks, 2007. (4)