Numerical Analysis

Douglas Faires, Youngstown State University, (Chair, 2012-2013)
Elizabeth Yanik, Emporia State University, (Chair, 2013-2015)
Graeme Fairweather, Executive Editor, Mathematical Reviews, AMS
Tim Sauer, George Mason University.

Introduction. Mathematical approximations have been used since ancient times to estimate solutions, but with the rise of digital computing the field of numerical analysis has become a discipline in its own right. Numerical analysts develop and study algorithms that provide approximate solutions to various types of numerical problems, and they analyze the accuracy, efficiency and robustness of these algorithms. As technology becomes ever more essential for the study of mathematics, learning algorithms that institution. Choosing between a course in which the students are expected to carry out low level coding in some computer language, and a course based on a software package, which allows the instructor to take a more high level conceptual view,

Course coverage

The following describe standard topics covered in a Numerical Analysis course or sequence. The topics after the preliminaries listed in the description for Numerical Analysis I are somewhat independent so there is considerable movement of topics between a first and second course, depending on the preparation of the students. For instance, if a Differential Equations course is a prerequisite for Numerical Analysis I, then approximations to differential equations are commonly studied in the first course. However, if students do not have this background, then the first course might omit this topic and include elementary techniques for solving linear systems, instead. The suggested time frames are only approximate, since time allotted for the topics in both Numerical Analysis I and II could vary significantly, depending on the amount of time allotted to student problem-solving and projects.

Numerical Analysis I

Preliminaries: (2 weeks) brief review of calculus material needed for the course; discussion of various types of computational error and ways to minimize it; software that is valuable for high accuracy approximations.

Solutions of a single nonlinear algebraic equation: (3-4 weeks) at a minimum, the bisection, secant and Newton's methods, with an analysis of the strengths and weaknesses of each; quadratic and linear convergence methods, and special techniques for improving convergence are often included.

Interpolation: (3 weeks) techniques for constructing polynomials and piecewise polynomials that agree with data or functional values at specified points---these include Lagrange polynomials in numerous forms with a discussion of error analysis; classical divided-difference methods are discussed as well as methods that approximate a function and its derivative(s) at various points; Hermite polynomials, cubic splines, and other interpolation techniques are frequently covered at this time or might be postponed to a second course.

Numerical differentiation and integration: (3 weeks) divided-difference methods are used to approximate derivatives of functions, and stability problems with these methods are considered; various numerical integration techniques are presented from the classical Newton-Cotes and Gaussian quadrature methods to the more sophisticated extrapolation and adaptive methods; multiple integration and improper integration might also be discussed.

Ordinary differential equations: (3-4 Weeks) first order initial-value problems and some theory concerning unique solutions might be discussed; classical methods based on Taylor series include Euler's method and, perhaps, the midpoint or modified Euler methods; Runge-Kutta methods are explored, with particular emphasis on the Runge-Kutta method of order 4; classical predictor-corrector methods based on the Adams techniques; if time permits, more sophisticated methods based on extrapolation, and/or variable-step size methods and adaptive methods with error control can be included; the course might also include approximations to systems of initialdescent or other techniques for obtaining starting values for more rapidly converging techniques; other modern techniques such as continuation methods could be discussed if time permits.

Boundary Value Problems and Partial Differential Equations: (3-4 weeks) shooting and finite difference methods for linear and nonlinear boundary value problems; finite difference techniques for partial differential equations with discussion of the accuracy difficulties for explicit difference methods; an introduction to the more modern techniques such as collocation and the finite-element method.

Possible textbook choices:

> Remark*: The presence of a text on this list is not meant to imply an endorsement of that text, nor is the absence of a particular text from the list meant to be an anti-endorsement. The texts are chosen to illustrate the sorts of texts that support various types of transitions courses. Please note that some of the books listed were written by the authors of this report.*

The five most commonly used texts used for Numerical Analysis courses are listed below:

Burden, R. L. and J. D. Faires, *Numerical Analysis*, 9th Ed., Brooks/Cole, Boston MA, 2011.

Sauer, T. D.