Operations Research Program Area Report

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Introduction

Mathematics departments that offer courses or concentrations in Operations Research (OR) often find that

Strategies. Putting modeling at center stage sometimes requires an instructor to leave the comfort zone of the traditional theorem-and-proof-driven mathematics course. Following are several recommended strategies for including modeling in an introductory OR course.

1. Word problems that teach students to distill a real-world scenario into mathematical language should be prominent in classroom activities and homework assignments.

2. Case studies and projects should be assigned to help students learn the process of formulating solving a real-world decision problems and communicating results to decision-makers.

3. Peer-reviewed articles in publications such as *OR/MS Today* and *Interfaces* should be assigned to demonstrate to students the wide applicability of OR and the role theory plays in making complex problems tractable.

A wide range of real-world scenarios can be modeled in an operations research course. These include public transportation network design, queueing systems, facility location, inventory management, airline scheduling, asset management, and even radiation therapy. After only a brief introduction to the field, students often quickly identify problems of personal relevance that can suggest class projects: dormitory room assignments, campus bookstore inventory management, course scheduling, etc.

Theory should be used as a tool for more effective application of OR techniques

Undergraduate OR courses should acquaint students with the essential reciprocity among theory, modeling, and computation in this field. In a traditional mathematics course, a primary objective could be to learn a mathematical theory and develop skills of mathematical proof. In an OR course, theory should serve applications. We offer some examples.

1. Duality theory plays an important role in sensitivity analysis of solutions to linear programming problems. Sensitivity analysis permits an operations researcher to determine and communicate the range of applicability of recommended solutions.

2. The shape of the feasible region and objective function (e.g., convex versus non-convex) in a mathematical program determines whether a local optimum is guaranteed to be a global optimum, and informs the sorts of algorithms that can be used.

3. The unimodularity property of the constraint matrix of network flow problems guarantees that network problems with integer data will have integer solutions to their linear programming relaxations. Integer programs that can be modeled in this way are therefore as easy to solve as linear programs.

4. In stochastic processes, the eigenvalues of the transition probability matrix of an ergodic Markov chain guarantee steady-state behavior that permits the analysis of queues.

It is less important that students prove many theorems rigorously than that they understand the statements and implications of a few fundamental theorems, and how they relate to the solution of real problems. Follow-on courses or graduate work can then fill in the details of the rich mathematical theory underlying OR methods.

Students need transferable familiarity with computational tools

Computation is indispensable to the practice of OR; enormous datasets and models containing thousands of variables and constraints are routine. A typical workflow requires using one type of software (say, R) to clean and summarize data, another (say, AMPL) to generate an optimization model from a set-based description, another (say, CPLEX) to solve the model, and perhaps yet another to display a solution graphically. All software has a short shelf-life, so graduates must be prepared to learn and integrate new tools as needed.

Following are some recommendations concerning computation.

1. Instructors should teach, and students should use, appropriate specialized software for each project or course: statistical or database software for data-intensive tasks, set-based modeling software for expressing optimization models, particular solvers for different families of constraint and objective functions.

2. Regarding Microsoft Excel: Students can build rich mathematical models, including simulations, decision trees, discrete dynamical systems, and linear and non-linear optimization models, using Excel and add-ins. Although it has drawbacks as a teaching tool, Excel has low barriers to entry for students in a first OR course, and is more readily available than specialized operations research software.

3. Computational tools can help students understanding theory, as when we use a MATLAB script or calculator program to do single pivots of the simplex algorithm. Reliable pivots are conducive to quick demonstrations of cycling in the simplex algorithm, or reviewing the connection between basic solutions and the geometry of the feasible region.

4. Students should learn the limits of computational tools and understand that software is not a black box that gives incontrovertible answers.

5. **Students pursuing a concentration or major in operations research should learn general programming skills (loops, conditional execution) using a widely-used language like C++, Python, Java, or MATLAB.**
   thinking about algorithms and optimality conditions. Students who can program are prepared to learn about computational complexity, which sometimes derails large models. Nevertheless, lack of programming skills should not rule out students who want to take only one or a few OR courses.

Curricula in OR should build a strong foundation in probability and statistics

Uncertainty permeates real-world decision-making, and large data sets are becoming ubiquitous. Without a strong foundation in probability and statistics, a student continuing in either graduate work or industry in OR will be at a severe disadvantage. Basic familiarity with probability models and stochastic processes permits students to study queuing theory, inventory management and other stochastic operations research topics. Statistics, with a focus on large-scale data1 19.114 Tm[(i)-4(z)(ed s)-3()11(at[(f)-4(ie)8T(cs)-3(, v recognition of common problem structures. But there is no substitute for solving a real problem in the messy real world. Students learn quickly that an unreasonable amount of simplification might be needed for a problem to fit neatly in the form of problems seen in class. They will experience first-hand the computational limitations that arise when solving problems with many constraints and variables and will be forced to think creatively to develop heuristics to solve them.

Real-world problem-solving opportunities can arise in several ways:

1. As a course project, students might identify problems of personal relevance to them or that arise in student life where an operations research approach could be useful. Examples include dorm room assignment, course scheduling, and dining hall inventory management.

2. As a senior capstone (e.g., a thesis or team project) experience, an organization can propose a problem needing an OR perspective; student(s) communicate regularly with liaison from the organization to ensure that the project is achieving the desired objectives.

3. As a summer res ithere i 01 10BT/TIETd Tm[1ise

We encourage math departments to offer OR courses that help students make meaning of the mathematics they learn. OR is fundamentally about modeling: translating and adapting real organizational problems (e.g., designing train schedules or radiation therapy treatment plans) into suitable mathematical models that can guide decision-making. Both theory — the body of beautiful mathematics that elucidates solution structures — and computation are indispensable in using those models to make recommendations to decision-makers. OR students should be encouraged to take creative approaches and to articulate assumptions when tackling under-specified problems, to learn computational tools, to know which theoretical results apply in particular problem settings, and to communicate about mathematics throughout the modeling process.