Topology

Erica Flapan, Pomona College (Chair)
Colin Adams, Williams College
Shea Burns, North Carolina AT&T State University

The inquiry-based topology course described below utilizes collaboration and independent in class.

Topic goals:

1. Metric spaces
2. Sequential compactness
3. Topological spaces
4. Connectedness and Path Connectedness
5. Compactness

Applied Topology

Applied Topology is intended to expose students with majors in mathematics or the sciences to basic topology and its recent applications. The prerequisite is linear algebra together with an introduction to proofs. However, a more advanced version of this course could be taught with a prerequisite of a one-semester course in undergraduate analysis.

Topic goals:

1. Open sets and topological spaces
2. Continuous functions and homeomorphisms
3. Metric spaces
4. Connectedness and compactness
5. Applications of topology

Knot Theory

Knot Theory is intended to expose students to ideas and proofs in visual mathematics. It is a good course for mathematics majo visual intuition and insight. However, it is also a good course for non-majors, since there are a lot of pictures and students can quickly get into interesting mathematics. It has the potential to draw new students into the major, as well as to give students the background they need to do undergraduate research projects. In addition, this is a good course for future teachers since it complements a traditional geometry course and could give students ideas for topics to bring in high school courses. The prerequisite for this course is the ability to handle mathematical proofs. This could come from a course in Linear Algebra, An Introduction to Proofs, Geometry, or Elementary Number Theory.

Topic goals:

1. Basic definitions, including knots, composition and Reidemeister moves
2. Tabulation of knots
3. Types of knots, including torus knots, satellite knots and braids
4. Surfaces and knots
5. Invariants of knots, including polynomials

Combinatorial Topology

Combinatorial Topology has a profile similar to that of Knot Theory; however, this course might be preferred by a department that would like a survey of several topological topics rather than a course that goes into more depth in Knot Theory.

Topic goals:

1. Topological equivalence
2. Orientability
3. Topology of surfaces
4. Examples of 3-manifolds
5. Basic knot theory

Appendix

Detailed Course Syllabi

Remark*: The presence of a text listed here is not meant to imply an endorsement of that text, nor is the absence of a particular text from the list meant to be an anti-endorsement. The texts are chosen to illustrate the sorts of texts that support the various types of Topology courses described by the authors. Please note that some of the books listed were written by the authors of this report.*

Title of course: Honors Topology
Credit hours/semester: 3 credit course
Target student audience: Junior/senior mathematics majors interested in preparing for graduate study in mathematics or closely related areas, such as theoretical physics.

Course description: This course covers topological spaces and metric spaces, including separation axioms, quotient spaces, compactness, connectedness, path connectedness, and homotopy, the Brouwer fixed point theorem, the fundamental group, covering spaces, and the classification of surfaces (if time permits).

Proposed prerequisites: One semester of undergraduate analysis, plus one semester of abstract algebra. Since the algebra will onm ts).

Most topics listed below should take 1-2 weeks to cover. Some topics, especially #s 4, 6, 9, 10, and 12 will each take about 2-3 weeks. The order of some topics may be switched, depending on the choice of textbook.

1. The notion of a topological space. Examples. Equivalent definitions. The case of a metric space.

2. Separation axioms, in particular Hausdorff and normal.

3. Quotient topologies and examples.

4. Compactness and sequential compactness. How compactness interacts with the Hausdorff axiom. The product of compact spaces is compact (finite Tychonoff). (Optional) Full Tychonoff assuming the axiom of choice.

5. Connectedness and path connectedness.

6. (Optional) Urysohn's Lemma and the Tietze extension theorem.

7. (Optional) More on metric spaces, the Arzelà-Ascoli theorem.

8. Homotopy, the fundamental group.

9. The Brouwer fixed point theorem and no retraction theorem.

10. Covering spaces and lifting of maps to covering spaces.

11. (Optional) Classification of surfaces (depending on time, perhaps omit the proof)

12. (Optional) Simplicial complexes and their topological realizations. Computing (combinatorially) the fundamental group of a simplicial complex.

13. (Optional) Simplicial homology as a (computable!) homotopy invariant. Connection $_1$ and H_1.

Possible Textbooks:

1. Armstrong, M. A., *Basic Topology*, Springer, 1983.
2. Crossley, M., *Essential Topology*, Springer, 2005.
3. Kosniowski, C., *A First Course in Algebraic Topology*, Cambridge University Press, 1980.
4. Munkres, J., *Topology*, Prentice Hall, 2000.
5. Runde, V., *A Taste of Topology,* Springer, 2005.
6. Shick, P. L., *Topology: Point-Set and Geometric*, Wiley, 2007.

Modes of delivery: This course is usually taught as a conventional lecture course, with regular problem sets making up a key part of the course. Classes from time to time should be spent discussing problems and having students participate in solving them.

Writing and use of technology: Students would be expected to write clear, rigorous proofs in their problem sets. The instructor might require students

Course Outline:

1. Metric spaces, open and closed sets (1 week)

2. Continuity and sequences in metric spaces, closure and interior (2 weeks)

3. Sequential compactness, the product metric (2 weeks)

4. Abstract topological spaces and continuity (1.5 weeks)

5. Bases, the subspace topology and the product topology (1.5 weeks)

6. Sequences, the Hausdorff property (2 weeks)

7. Connectedness and path connectedness (2 weeks)

8. Compactness (1-3 weeks)

Optional topics:

9. The quotient topology
10. Complete metric spaces

Possible texts: This course is most naturally run without a textbook. However, there are a number of sources for well-prepared materials that can be used to support such a course.

Sample worksheet sets for this specific course are available from Dick Canary (canary@umich.edu).

For other possible IBL Topology courses, the *Journal of Inquiry-based learning in Mathematics*, JIBLM, is a source of peer-reviewed, class-tested, inquiry-based notes.

learn how to construct and critique rigorous mathematical arguments. The students also gain experience presenting mathematical results.

In addition, the course satisfies the following general principles enumerated by the CUPM:

What should we teach? - Thinking

This course should help students develop effective thinking skills. The activities of the

> Approach problem-solving with curiosity and creativity, with a willingness to try multiple approaches, persist in the face of difficulties, asses the correctness of solutions, explore examples, pose questions, and devise and test conjectures.
> Communicate mathematical ideas with clarity and coherence through writing and speaking.
> Function as an independent mathematical thinker and learner.

This course should present the beauty and joy of mathematics.

How should we teach? - How people learn

This course should make mathematics meaningful by using an interactive process in which students participate in the development of new concepts, questions, and answers. Students should regularly develop mathematical ideas and explain their ideas both by writing and by speaking.

Title of course: Applied Topology
Credit hours/semester: 3 credit hours
Target student audience: Sophomores, juniors and seniors with majors in either mathematics or the sciences.

Course description: This course is a basic introduction to topology and topological spaces, motivated by applications of topology that have proliferated in recent years. The course covers the standard topics in an introductory point-set Topology course, including topological spaces, open and closed sets, bases, interior, closure, limit points and boundary of subsets, continuous functions, homeomorphisms, connectedness and compactness, as well as a range of applications.

Proposed prerequisites: The prerequisite is linear algebra together with an introduction to proofs. However, a more advanced version of this course could be taught with a prerequisite of a one-semester course in undergraduate analysis.

How the course might fit into a program of study: This course is intended as an elective course in mathematics, appropriate for students intending to do graduate work in mathematics. However, it could also be useful to students intending to pursue advanced degrees in subjects to which the applications are geared.

Course Outline:

1. Preliminaries : This includes a basic introduction to what topology is, perhaps a little history, and then a basic set theory notation, functions, sequences, equivalence relations,

Euclidean n-space and the difference between countable and uncountable sets.

2. Topological spaces: This includes the definition of a topological space, open and closed subsets, and bases for a topological space. Applications can include digital image displays and RNA folding.
3. Interior, closure, limit points and boundary of a set: This includes how the concepts of interior, closure, and limit points relate to one another, and the definition of the boundary of a subset in a topological space. Applications can include Geographic Information Systems.
4. New topological spaces: This includes the subspace topology, the product topology, and

Modes of delivery: This course is typically taught in a lecture format with problem sets

Proposed prerequisites: The prerequisite for this course is the ability to handle mathematical proofs. This could come from a course in linear algebra, a transition to proofs, geometry, or elementary number theory.

How the course might fit into a program of study: This course is intended as an elementary elective course in mathematics. It can serve well as a bridge between an introductory proofs course and the more substantial analysis and/or algebra courses. It is a good course for n and insight. However, it is also a good course for non-majors, since there are a lot of pictures and students can quickly get into interesting mathematics. It has the potential to draw new students into the major, as well as to give students the background they need to do undergraduate research projects. In addition, this is a good course for future teachers since it complements a traditional geometry course and could give students ideas for topics to bring into the high school classroom.

Course Outline:

1. Introduction: A basic introduction to knot theory including some history, composition of knots, Reidemeister moves, links, tricoloration, linking number. Stick number can be introduced as a simple physical invariant that has applications to synthesizing knotted molecules.
2. Tabulating knots: Simple notations for knots, including Dowker notation and Conway notation.
3. Invariants of knots: The traditional invariants of knots, including the crossing number, the bridge number, and the unknotting number of knots.
4. Surfaces and knots: The theory of surfaces with and without boundary. If so desired, one can present the classification of surfaces. Seifert surfaces, the genus of a knot and the proof that genus is additive under composition. Proof that the composition of two nontrivial knots cannot be trivial.
5. Types of knots: Important categories of knots: can include torus knots, satellite knots, hyperbolic knots, 2-bridge knots, alternating knots and almost alternating knots.
6. Polynomials of knots: Introduction to the various polynomials of knots, including the Jones
   and its application to prove that every reduced alternating projection of a knot must have the same crossing number, the Alexander and Homflypt polynomials, and facts about amphicheirality of knots.
7. Biology and chemistry and knots: Applications of knot theory to DNA and DNA knotting, synthesis of knotted molecules.

Optional topics:

Possible textbooks:

1. Adams, C., *The Knot Book*, AMS, 2004.
2. Cromwell, P., *Knots and Links*, Cambridge University Press, 2004.
3. Gilbert, N. and T. Porter, *Knots and Surfaces*, Oxford University Press, 1996.
4. Livingston, C., *Knot Theory*, MAA, 1993.
5. Murasugi, K., *Knot theory and its Applications*, Birkhauser, 1996.

Supplementary material can be obtained from:

6. Flapan, E., *When Topology Meets Chemistry*: *A Topological Look at Molecular Chirality*, MAA and Cambridge University Press, 2000.
7. Kauffman, L., *On Knots*, Princeton University Press, 1987.

Modes of delivery: This course is typically taught in a lecture format with problem sets throughout the course, and several exams. In class, faculty can involve students in conjecturing what might be true, and include exercises that ask students to do this.

Writing and use of technology: It is natural to include a writing component in the course. The instructor collects a set of research papers that are relevant to the covered material and that are readable by students with this level .1 TmabcudtP /Md11(.1 0 0 1 173.3 517.8Fs a se)4(t)]TJETBT 0 1 279.7ETB

This course should present the beauty and joy of mathematics.

How should we teach? - How people learn